A novel method of Medical image denoising using bilateral and NLm filtering.

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Abstract— **One of the bottleneck in medical imaging is due to low Signal-to-Noise Ratio(SNR) which requires long and repeated acquisition of the same subject to reduce noise and blur. To obtain a high SNR image without lengthy repeated scans, post-processing of data such as denoising plays a critical role. Bilateral and Non-Local means (NLm) filtering are commonly used procedures for medical image denoising. In this paper we propose a new thresholding scheme to wavelet and contourlet based denoising by introducing a scaling factor to universal threshold. Also we propose the aforementioned novel contourlet thresholding scheme as a pre-processing step on bilateral filtering and NLm denoising. Simulation results show that the proposed single entity comprising the novel preprocessing step and bilateral or NLm denoising is superior in terms of PSNR and perceptual quality compared to Bilateral filtering or NLm denoising used alone.**

Keywords—Denoising,Contourlet transform,Bilateral filtering,Non Local means denoisng,wavelet transform,visushrink.

I. INTRODUCTION

The rapid development of advanced medical imaging technologies such as Magnetic Resonance Imaging(MRI), Positron Emitted Tomography (PET) and Computer Tomography (CT) have provided us an unprecedented way to diagnose illness in-vivo and non-invasively. Some of the most advanced techniques based on this imaging modality remain in the research stage but never reach routine clinical usage. One of the bottlenecks is due to low Signal-to-Noise Ratio (SNR) which requires long and repeated acquisition of the same subject to reduce noise and blur. For example, a high SNR Diffusion Tensor Imaging (DTI) dataset requires an hour of data Acquisition[1]. A high SNR of High-Angular-Resolution Diffusion Imaging (HARDI) data can take 13 hours [2]. To recover high SNR image from noisy and blurry image without lengthy repeated scans, post-processing of data plays a critical role in two aspects: (1) automatic denoising and deblurring algorithms can restore the data computationally to achieve a quality that might take much longer otherwise. (2) computational object segmentation techniques can perform intelligent extraction of data directly and automatically from noisy observations.

In medical imaging we often face a relatively low SNR with good contrast, or a low contrast with good SNR.

Fortunately the human visual system is highly effective in recognizing structures even in the presence of a considerable amount of noise. But if the SNR is too small or the contrast is too low it becomes very difficult to detect anatomical structures. A definition of overall image quality includes physical and perceptual criteria. Furthermore, it largely depends on specific diagnostic tasks. In some cases a high spatial resolution and a high contrast are required, whereas in other cases more perceptual criteria may be favoured. For a visual analysis of medical images, the clarity of details which mainly comprises edge information and the object visibility are important.

Noise mostly occurs in medical images are Additive White Gaussian noise (AWGN), salt and pepper noise[3], speckle noise[4][5][6] etc. There are two basic approaches to image denoising, spatial filtering methods and transform domain filtering methods. The filtering does not respect region boundaries or small structures, and the resulting images appear blurry and diffused. Many researchers have used spatial as well as transform domain filtering in medical imaging[7-10]

When developing a filtering method for medical image data, image degradation by blurring or artifacts resulting from a filtering scheme is not acceptable. The following two criteria should ideally be fulfilled:

- Minimize information loss by preserving object boundaries and detailed structures.

- Efficiently remove noise in regions of homogeneous physical properties.

Recent developments based on bilateral filtering [11] overcome the major drawbacks of conventional spatial filtering, and significantly improve image quality while satisfying the main criteria 1 stated above. Bilateral filtering is one of the examples of non-linear non-iterative filtering. It combines domain and range filters simultaneously. It preserves edge information while denoising. Many medical image denoisng algorithm have been proposed using bilateral filtering[6,7]. However it does not give satisfactory results since real gray levels are polluted seriously and the range filter cannot work properly. This would lead to bring side effect to the denoising results like a polishing look to denoised image. For example, image's tissue regions or brain's grooves may be

weakened. This phenomenon is referred as propagation of noise (PoN) [11].

As an extension of bilateral filtering, Buades et al.[12] had put forward Non-Local means image denoising which utilizes structural similarity. Generally speaking, the information in a natural image is redundant to some extent. NL means image denoising algorithm takes full advantage of image redundancy. The basic idea is that images contain repeated structures, and averaging them will reduce the (random) noise. It is an efficient denoising method with the ability to result in proper restoration of the slowly varying signals in homogeneous tissue regions while strongly preserving the tissue boundaries. NLm has been used in medical image denoising in many works [13]. However, the NLM filter may suffer from potential limitations since the calculation for similarity weights is performed in a full-space of neighborhood. Specifically, the accuracy of the similarity weights will be affected by noise.

Donoho and Johnstone proposed an innovative nonlinear denoising scheme in transform domain VisuShrink [14] which thresholds the wavelet detail coefficients for one dimension (1-D) signals. This scheme is simple and efficient. More recently, wavelet based filters have been applied successfully to MR image denoising [15,10,4]. Donoho and Johnstone have shown that VisuShrink results in an estimate asymptotically optimal in the minimax sense. When images are denoised by VisuShrink, in many cases it can outperform the classic linear Wiener filtering, especially for the images with a low peak signal-to-noise ratio (PSNR). However, it is also well known that universal thresholding yields overly smoothed images. This is because its threshold choice can be unwarrantedly large due to its dependence on the number of samples. Many researchers have made claim that the universal threshold is not the perfect threshold and performance can vary around this threshold [4,6].

Minh N. Do and Martin Vetterli put forward a new dimension in image transform named contourlet transform[16,17],which proved to be much better than the wavelet transform in conserving edges and line details. The improvement in approximation by contourlets based on keeping the most significant coefficients will directly lead to improvements in denoising. A simple thresholding scheme applied on the contourlet transform is more effective in removing the noise than it is for the wavelet transform[17]. A number of works have been proposed in image denoising using contourlet transform recently. Atif et al.[18] used the same thresholding scheme which is used in wavelets to contourlet denoising, to substantiate the claim made in paper[17]. The problem is that directly using the universal threshold which suites well in wavelet domain is not suitable in contourlet domain, since the number of elements in transform coefficients for contourlet is much higher than wavelet coefficients.

In this paper we propose a novel technique to medical images corrupted by AWGN. This paper puts forward three contributions

- As the universal thresholding is an estimate which is asymptotically optimal in the minmax sense, we introduce a scaling parameter to the universal threshold by extensive simulation and regression of data with wide range of standard medical images as well as

natural images of different size and corrupted by different noise variances.

- We have also extended this idea to contourlet transformation with a new scaling factor and the same universal thresholding.

- Proposed contourlet thresholding is performed as a pre-processing step prior to bilateral and NLm filtering, which can give significant improvement in PSNR as well as visual quality from bilateral and NLm filter used alone.

The paper is organized as follows. Section II introduces Bilateral filtering, Non Local mean filtering, wavelet shrinkage and contourlet transform. In Section III, Proposed Method is discussed. The Section IV presents the experimental results and discussion. The concluding remarks are given in Section V.

II. BACKGROUND

A. Bilateral Filtering

The bilateral filter is a nonlinear filter proposed by Tomasi and Manduchi to smooth images[11].The key idea of the bilateral filter is that for a pixel to influence another pixel, it should not only occupy a nearby location but also have a similar value. The idea underlying bilateral filtering is to do in the range of an image what traditional filters do in its domain. Two pixels can be close to one another, that is, occupy nearby spatial location, or they can be similar to one another, that is, have nearby values, possibly in a perceptually meaningful fashion. Closeness refers to vicinity in the domain, similarity to vicinity in the range. Traditional filtering is domain filtering, and enforces closeness by weighting pixel values with coefficients that fall off with distance. Similarly, range filtering is defined, which averages image values with weights that decay with dissimilarity. Range filters are nonlinear because their weights depend on image intensity or color. The bilateral filter is also non-iterative, i.e. it achieves satisfying results with only a single pass. This makes the filter's parameters relatively intuitive since their action does not depend on the cumulated effects of several iterations. Computationally, they are no more complex than standard non-separable filters. Most importantly, they preserve edges.

The weight assigned to each neighbour decreases with both the distance in the image plane (the spatial domain S) and the distance on the intensity axis (the range domain R). Using a Gaussian G_σ as a decreasing function, and considering a gray level image I, the BF[I] of the bilateral filter is defined by:

$$BF[I]_{\mathbf{p}} = \frac{1}{W_{\mathbf{p}}} \sum_{\mathbf{q} \in S} G_{\sigma_s}\left(\|\mathbf{p}-\mathbf{q}\|\right) G_{\sigma_r}\left(|I_{\mathbf{p}}-I_{\mathbf{q}}|\right) I_{\mathbf{q}} \qquad (1)$$

- I_p = value of image I at position: $p = (p_x, p_y)$

- F [I] = output of filter F applied to image I

The parameter σ_s defines the size of the spatial neighborhood used to filter a pixel, and σ_r controls how much an adjacent pixel is down weighted because of the intensity difference. W_p normalizes the sum of the weights.

On the other hand, the bilateral filter is nonlinear and its evaluation is computationally expensive since traditional acceleration, such as performing convolution after an FFT, are not applicable. Many fast algorithm have been formulated for bilateral filtering [19,20], one of the recent algorithm is formulated by Choudhary et al. [21] .

B. Non-Local Mean filtering

Buades et al. [12] developed a Non-Local means (NLm) image denoising algorithm which takes full advantage of image redundancy. The basic idea is that images contain repeated structures, and averaging them will reduce the (random) noise. The NL-means filter is an evolution of the Yaroslavsky filter [22] which averages similar image pixels defined according to their local intensity similarity. The main difference between the NLmeans and this filter is that the similarity between pixels has been made more robust to noise by using a region comparison, rather than pixel comparison and also that matching patterns are not restricted to be local. That is, pixels far from the pixel being filtered are not penalized. Given an image Y the filtered value at a point i using the NL-means method is computed as a weighted average of neighbouring pixels N_i in the image following this formula:

$$NL[Y(i)]_\mathbf{p} = \sum_{j\in Ni} w(i,j)Y(j) \qquad (2)$$

$$0 \le w(i,j) \le 1 \quad \text{and} \quad \sum_{j\in Ni} w(i,j) = 1$$

where i is point being filtering and j represents any other image pixel. The weights $w(i, j)$ are based on the similarity between the neighborhoods N_i and N_j of pixels i and j. N_i is defined as a square neighbourhood window indices centered around pixel i. Theoretically, noise filtering performed must be considered as an estimation task. Since the estimation task of weights $w(i,j)$ are computationally expensive, a large number of fast methods have been developed[23,24].

C. Wavelet shrinkage

Wavelet transform shows localization in both time and frequency and hence it has proved itself to be an efficient tool for a number of image processing applications including noise removal[14,15].The methods based on wavelet representations yield very simple algorithms that are often more powerful and easy to work with than traditional methods of function estimation. *Visushrink* consists of decomposing the observed signal into wavelets and use thresholds to select the coefficients, from which a signal is synthesized. The idea behind wavelet shrinkage is that Wavelet is such a basis because exceptional event generates identifiable exceptional coefficients due to its good localization property in both spatial and frequency domain [5]. But considering noise, as long as it does not generate exceptions, additive white Gaussian noise after applying WT is still AWGN. A large fraction of any L number of random data array with zero mean and variance σ^2, will be smaller than the universal threshold T with high probability; the probability approaching 1 as L increases to infinity[14] where

$$T = \sigma\sqrt{2\log_e L} \qquad (3)$$

However, VisuShrink yields overly smoothed images. This is because its threshold choice can be unwarrantedly large due to its dependence on the number of samples and universal thresholding is an estimate which is asymptotically optimal in the minmax sense.

D. Contourlet Transform

The contourlet transform is a new geometrical image-based transform, which is recently introduced by Do and Vetterli [17]. In contourlet transform, the Laplacian pyramid [25] is first used to capture the point discontinuities, then it is followed by a directional filter bank [26] to link point discontinuities into linear structures. The overall result is an image expansion using elementary images like contour segments, called contourlet transform, which is implemented by a pyramidal directional filter bank. The Laplacian pyramid (LP) is used to decompose an image into a number of radial subbands, and the directional filter banks (DFB) decomposes each LP detail sub-band into a number of directional subbands.
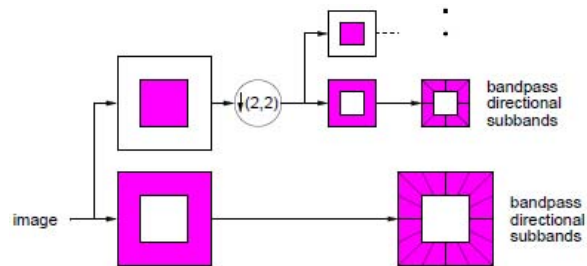

Figure 1.   Contourlet implementation using directional filterbank and pyramidal filter banks.

The improvement in approximation by contourlets based on keeping the most significant coefficients will directly lead to improvements in applications, including compression, denoising, and feature extraction. As an example, for image denoising, random noise will generate significant wavelet coefficients just like true edges, but is less likely to generate significant contourlet coefficients. Consequently, a simple thresholding scheme applied on the contourlet transform is more effective in removing the noise than it is for the wavelet transform[17].

III.   PROPOSED METHOD

Wavelet thresholding which is a signal estimation technique that exploits the capabilities of wavelet transform for signal denoising, removes noise by killing coefficients that are insignificant relative to some threshold. It turns out to be simple and effective, depends heavily on the choice of a thresholding parameter and the choice of this threshold determines, to a great extent, the efficacy of denoising. Researchers have developed various techniques for choosing denoising parameters and so far there is no "best" universal threshold determination technique. As the universal thresholding is an estimate which is asymptotically optimal in the minmax sense and universal threshold as derived by Donoho is hundred percent effective only when number of pixels in an image tends to infinity, which is an impractical situation. Considering this delicate argument we introduce a scaling parameter to the universal threshold which is suitable

for medical images whose number of pixels are finite, by extensive simulation and tabulation with different standard medical images of different size and corrupted by different noise variances. The experimental procedure we adopted is taking an arbitrary image of known size and corrupted by a known noise variance. Then we introduced scaling factors for the universal threshold and selected the scaling factor corresponding to the highest PSNR. This procedure is repeated for large set of images of different size and corrupted by different noise variance. The compact equation of scaling parameter is obtained by linear regression of the best scaling parameter as a function of noise variance and image size.

For wavelet thresholding, tabulation and regression with different sets of medical images as well as natural images give the threshold λ_w to be

$$\lambda_w = 3.944 \times 10^{-11} \, S^2 - 5.5285 \times 10^{-6} \, S + 0.6022 \qquad (4)$$

where S is a function of noise variance and number of image pixels, which is given as

$$S = \sigma \times \sqrt{N}$$

N is the number of pixels of the image under consideration and σ is the standard deviation of noise. So the new threshold is $T_w = \lambda_w \times T$.

Same set of experiments done on wavelet thresholding is performed with contourlet transformation. For contourlet thresholding experimental measurements gives the threshold λ_c to be

$$\lambda_c = 3.944 \times 10^{-11} \, S^2 - 5.5285 \times 10^{-6} \, S + 0.5522 \qquad (5)$$

where S is a function of noise variance and number of image pixels, which is given as

$$S = \sigma \times \sqrt{N}$$

N is the number of pixels of the image under consideration and σ is the standard deviation of noise. So the new threshold is $T_c = \lambda_c \times T$. The reason for the reduction in scaling factor is that, in contourlet transform, number of transform coefficients are large compared to the wavelet(in which number of transform coefficients equal to image pixels), a similar reduction in weight of coefficients take place in contourlet domain, so as to distribute values over all contourlet coefficients. Simulation results show that proposed contourlet thresholding and wavelet thresholding outperforms the standard Visushrink.

Bilateral filtering in which real homogeneous gray levels corrupted by noise is polluted significantly, and fails to efficiently remove noise in regions of homogeneous physical properties. In homogeneous regions in which noise strike, bilateral filtering most likely perform range filtering much greater than spatial filtering. So the effect of noise is retained as if it were an edge information of the image. If contourlet denoising which is computationally efficient and having negligible execution time is done before bilateral filtering, noise in homogeneous regions which is present as un-exceptions can be removed efficiently retaining the edge information as well as texture. Simulation results show that preprocessing with proposed contourlet denoising is superior to proposed wavelet thresholding. The reason is that the only aim of the denoising prior to bilateral filtering is to remove discrepancies in homogeneous regions, but retaining edge information and texture details as such. But wavelet transform using Visushrink smoothens the image[15] thereby degrading performance of bilateral filtering. But contourlet thresholding is good in retaining directions and edge features.

The NLM filter may suffer from potential limitations since the calculation for similarity weights is performed in a full-space of neighborhood. Specifically, the accuracy of the similarity weights will be affected by noise. Performing contourlet denoising prior to NLM filtering can compensate noise, which can increase the accuracy of similarity weights in the full space of neighbourhood and hence increase the performance of NLM filtering. Preprocessing with proposed contourlet thresholding give better results compared to proposed wavelet thresholding because contourlet denoising is much efficient in retaining edges and texture information and hence similarity weights after thresholding.

IV. SIMULATION RESULTS

A set of two gray scale images, MRI image of human brain and CT image of abdomen, shown in Fig. 2, were selected for our simulations. Each image has a size of 512 x 512 and 1024 x 1024 pixels respectively with 256 shades of gray, i.e. 8 bits per pixel. The evaluation criteria are based upon quantitative measure of Peak signal to noise ratio. These measures, as such, do not represent human perception of the images, i.e.visual quality of the images. It is very difficult to guage denoised images mathematically on visual quality as human vision system is not only highly subjective but also exhibit different tolerance levels to various types of noises. Thus authentication of denoising method requires both quantitative measure and qualitative inspection of image. In our work, we apply both criteria; first the PSNR and the image is also viewed for visual acceptance.

a   b

Fig. 2 (a) CT image of abdomen(CT) (b) MRI image of human brain(MRI).

Table I gives the comparison of calculated PSNR of images reconstructed utilizing various thresholding. Fig. 3 gives the corresponding images.

Table II gives PSNR comparison of image denoising using standard Bilateral filtering, standard non local mean filtering, and the proposed methods. Fig. 4 gives the corresponding images.
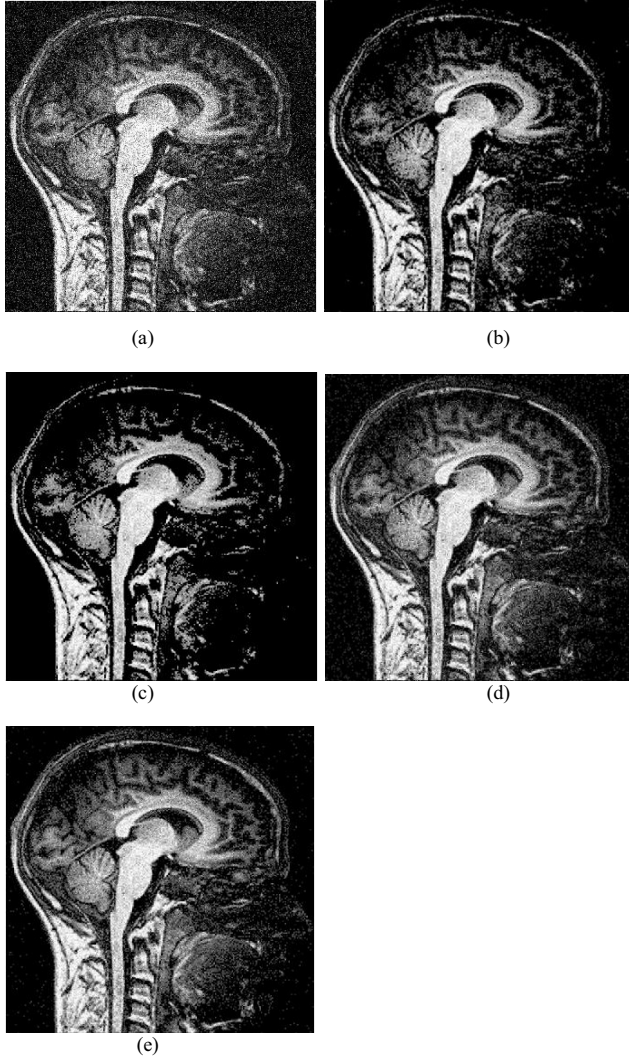
Fig. 3 Comparing threshold performance (a) Noisy image (σ = 40)
(b)universal thresholding with wavelet (c) universal thresholding with
contourlet (d)Proposed thresholding with wavelet (e)Proposed thresholding
with contourlet.

TABLE I. COMPARING THRESHOLD PERFORMANCE (PSNR).

Test image	Wavelet Universal Threshold	Contourlet Universal Threshold	Wavelet proposed Threshold	Contourlet proposed threshold
σ =30				
CT	19.2590	18.0033	**22.7820**	**23.2653**
MRI	20.8224	19.4452	**23.7486**	**23.8390**
σ =40				
CT	16.1492	14.9014	**20.2541**	**20.7107**
MRI	18.2044	16.7866	**21.4390**	**21.5036**
σ =50				
CT	13.8143	12.6572	**18.4467**	**18.6677**
MRI	16.2471	14.8722	**19.7023**	**19.8100**

TABLE II. COMPARING PROPOSED METHOD AND STANDARD MEDICAL IMAGE FILTERING (PSNR).

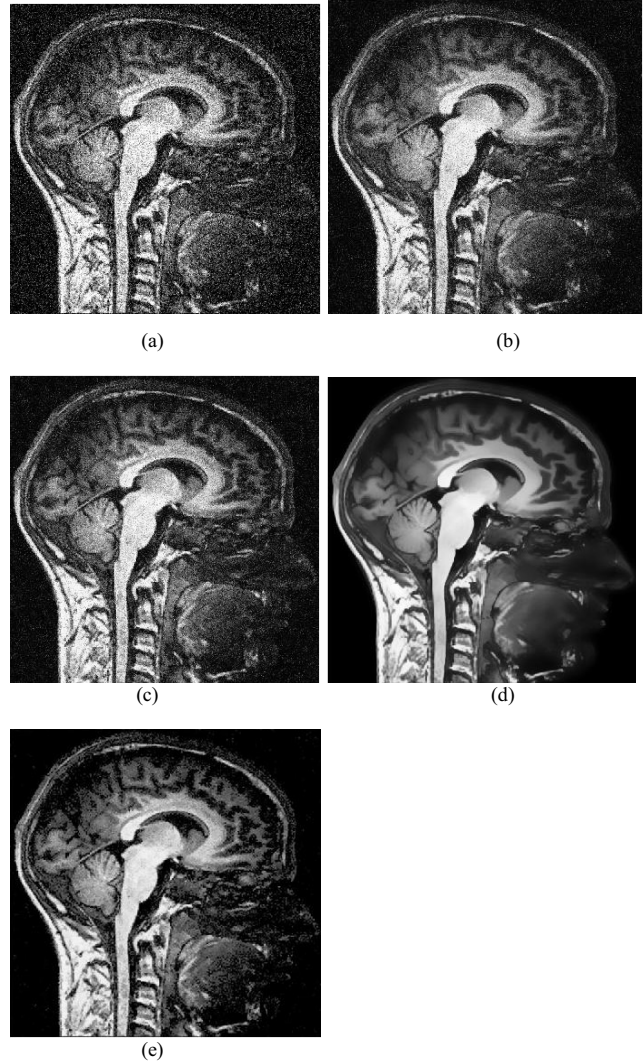| Test image	Bilateral filtering	NLm filtering	Proposed preprocessing prior to bilateral filtering	Proposed preprocessing prior to NLm filtering
σ = 30				
CT	23.8733	23.9525	**30.0319**	**30.3344**
MRI	22.5029	23.5457	**25.8689**	**26.2227**
σ = 40				
CT	20.0548	19.8352	**26.1260**	**25.1537**
MRI	19.7281	20.1621	**23.5916**	**23.5454**
σ = 50				
CT	17.3835	17.1381	**22.9191**	**22.0310**
MRI	17.4913	17.6718	**21.6140**	**21.4496**

Fig. 4 Comparing performance (a) Noisy image (σ = 40) (b)bilateral filtering
(c) NLm filtering (d)Proposed preprocessing with bilateral filtering
(e)Proposed preprocessing with NLm filtering.

It can be seen that, there is a significant improvement in PSNR values in the proposed threshold compared to the universal thresholding. Also by incorporating proposed contourlet thresholding prior to bilateral filtering and non-local mean filtering gives improved PSNR. Visually it's evident that the Proposed preprocessing with NLm filtering gives much more visual information, than with bilateral filtering even though pre-processing with bilateral filtering give good PSNR. The reason is that bilateral filtering incorporates a spatial filter apart from range filter which gives a smoothing effect.

V. CONCLUSION AND FURTHER WORKS

In this work we propose a novel preprocessing scheme for bilateral filtering and NLm filtering for medical image denoisning. We used contourlet transform, as a directional non-separable transform, to retain image textures and edges even after thresholding. A new threshold for denoising is devised using simulation results both in contourlet and wavelet space. The method is tested on CT abdomen and brain MR images. It is also compared with universal threshold. It is shown that for many noisy images, universal threshold based on wavelet is not a proper choice. Instead we can use contourlet based methods which can retain edges in the image much better than wavelet thresholding. We have shown that by using contourlet thresholding as a preprocessing step prior to bilateral filtering and Non local means filtering we can effectively denoise the image and also significantly increase the PSNR and perceptual quality.

In general as a future forethought any denoising scheme whose performance very much depend on spatial or structural features, preprocessing with proposed contourlet thresholding happens to be a better candidate in retaining the spatial or structural features and hence enhance the performance of aforementioned denosing scheme. Also it is also possible to device a similar thresholding scheme related with a particular application by simulation and regression with large sets of data.

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