LINEAR ALGEBRA FOR AI AND ML (AI61003)

More tutorials

Compensatory lectures

Tutorial and Programming assignment

Class test on Nov 08

Announcements

Course Logistics - Second Half

Instructor: Prabhat Kumar Mishra

Teaching Assistants: Asim Manna, Deepayan Chakraborty and Raj Kishore

Syllabus:

1. Least squares and constrained least squares

References: Part III, *Introduction to Applied Linear Algebra: Vectors, Matrices, and Least Squares*. Boyd and Vandenberghe. Cambridge University Press

2. Gradient descent and coordinate descent

Reference: Chap 5, *Statistical learning with sparsity*, Hastie, T., Tibshirani, R. and Wainwright, M., CRC Press.

3. Matrix approximation and completion

Reference: Chap 7, *Statistical learning with sparsity*, Hastie, T., Tibshirani, R. and Wainwright, M., CRC Press.

Theory + Tutorials

Time: Wed (10:00-10:55) , Thurs (09:00-09:55) , Fri (11:00-12:55)

Venue: NR123

Grading policy

Class Test-2 (10%)

Endsem (40%)

Programming (5%)

Bonus: For asking questions

Slides for the second half (pkmishra)

Lecture 01 & 02

Lecture 03 & 04

Lecture 05, 06 & 07

Lecture 08, 09 & 10

Tutorial_LS.ipynb

Tutorial_optimization.ipynb

Assignment

Lecture 11

Lecture 12

Lecture 13

Course Logistics - First Half

Course Logistics

Teachers: Prof. Mahesh Mohan M R
Ph.D Teaching Assistants: Asim Manna and Deepayan Chakraborty
References
1. Linear Algebra and Learning From Data, Gilbert Strang, Wellesley Cambride Press (available online)
2. Introduction to Linear Algebra, Gilbert Strang, Fourth Edition, Wellesley Cambride Press (available online)
3. Introduction to Applied Linear Algebra: Vectors, Matrices, and Least Squares. Boyd and Vandenberghe. Cambridge University Press (available online)
4. Linear Algebra, Friedberg, Insel, and Spence, Fourth Edition
Theory + Tutorials
• Wed (10:00-10:55) , Thurs (09:00-09:55) , Fri (11:00-12:55)
• Evaluation: Class Test-1, Midsem, Class Test-2, and Endsem (10%+30%+10%+40%)
• Class Tests 1 and 2 predominantly cover Tutorial questions and Reading assignments
Programming
• Two Assignments - one pre-midsem and one post-midsem (will be announced) (5% + 5%)
Office Hours: Pre-midsem time - Fridays 3-4 PM based on appointments
Plagiarism: No tolerance policy. Binary marking (both parties).
Attendance: Compulsory to avoid deregistration.

Logistics

Motivation: Linear Algebra for AI and ML

A brief history of AI and Linear Algebra (LA)
Units of LA: Scalar, Vector, Matrix, and Tensors
Four Stages of Neural Network Training
Stage 1: Data Representation and Processing
Stage 2: Neural Network Modelling
Stage 3: Loss Functions
Stage 4: Optimization
Overview of topics based on Four Stages

Motivation and Overview

Reading Assignment: Pages **III** - **V** of *LA and Learning from Data* (first reference book)

Title: Deep Learning and Neural Nets

Stage 1: Data Representation and Processing

1. Part 1 -- Basis based Dimensionality Reduction
- Creating Basis from a set of vectors
  - Column Space and Independence
  - Solution exists or not in Linear Equations
- Compressing the Data using Basis
  - Solution of Linear Equations
  - Reconstruction using Matrix-Matrix Multiplication
- Test data not in Column Space
  - Least Square Estimation
2. Part 2 -- Orthonormal Basis based Dimensionality Reduction
- Constructing Orthornormal Basis from a set of basis vectors (Gram Schmidt Method)
  - Inner product, Angle, orthonormal vectors
- Compressing the Data using Basis
  - Simple inner product to find the coefficients
- Test data not in Column Space
  - Simplified Least Square Estimation (without the need of inverse)
3. Part 3 -- PCA and SVD
- PCA Method: For lossy compression beyond what rank-based reduction permits
  - Covariance matrix -- Energy compaction efficiency and Compression efficiency
  - Eigen values and Eigen Vectors, and their computation
  - Symmetric Matrices have real Eigen values and orthonormal Eigen vectors
  - Lossy compression via Inner product of Eigen vectors with highest Eigen values (principal components)
- SVD Method: For reducing the rank of the data (so as to use Basis based dimensionality reduction)
  - A = U DV^T, where U and V are orthonormal matrix and D is a diagonal matrix with singular values
  - Computation of U, V, and D
  - Number of singular values = Rank of A
  - Rank reduction is via making smaller singular values to zero
  - Inverse and Pseudo Inverse (Right Inverse and Left Inverse)
  - Condition Number of a Linear System of Equations
4. Applications (Face Identification via PCA and Extreme MultiLabel Classification)

Data_Part1

Reading Assignment: Sec 1.1-1.3; Secs 5.1-5.2; Sec 8.3, Secs 12.1-12.2 of Introduction to Applied *LA* (third reference book)

Contents: Vector Arithmetics, Basis and Linear Independence, Linear Equations, Least Squares Solution

Tutorial Questions: Qn 1.2, 1.18, 1.20, 5.4, Examples in Sec 12.4 of Introduction to Applied *LA* (third reference book)

Qns 24, 25, 27 (page no: 65-66) of Introduction to Linear Algebra (second reference book)

Data_Part2

Reading Assignment: Sec 1.4, Secs 5.3-5.4 of Introduction to Applied *LA* (third reference book)

Sec 4.4 of Introduction to Linear Algebra (second reference book)

Contents: Inner Product, Orthonormal vectors, Gram Schmidt algorithm, QR decomposition

Tutorial Questions: Qn 1.11, 5.5, 5.6 of Introduction to Applied *LA* (third reference book)

Data_Part3

Reading Assignment: Sec V.1 and V.4 of LA and Learning From Data (first reference book)

Sec 6.1, 6.2, 6.4, page 403, 9.2 of Introduction to Linear Algebra (second reference book)

Contents: Mean, Variance and Covariance Matrix; PCA and SVD, Condition Number

Tutorial Questions: Qn 2, 3, 4, 6, 9 (page 293); Qn 1, 2, 6 (page 307); Qn 6, 7,8, 9, 11, 12 (page 372); Qn 7.3A, 6, 7, 10, 11 (page 407) of Introduction to Linear Algebra (second reference book)

Applications

Tutorial-1

Stage 2: Neural Network Modelling

Linear Transformations (additivity and homogeneity properties)
Inner product and its relation to Neural Network
Cauchy Schwarz Inequality
Linear Transformation Matrix and its computation
Isomorphic Transformation (one-to-one and onto)
Composition of Linear Transformations and resultant Matrix
Change of Basis as a Linear Transformation
Computationally Efficient Transformation via Change of Basis
Neural Network Modelling using Linear Transformation
Perceptron (XOR problem)
Multi-layer Perceptron (Why we need non-linearities)
Convolutional Neural Network (Why we need convolution layers)
Transformer (Attention layers)

Linear_transformation

Reading Assignment: Chapter 7 of Introduction to Linear Algebra (second reference book)

Tutorial Questions: Qns 1-12 (page 380); Qn 5, 6, 7 (page 395); Qn 36 (page 398) of Introduction to Linear Algebra (second reference book)

Tutorial-2

Stage 3: Loss Functions

Norms and properties (positivity, homogeneity and triangular inequality)
Proof of Norm properties using Inner Product properties
Different Norms: L1, L2, Chebyshev, and L-infinity Norms
How Norms act as loss function in AI/ML

Loss Functions

Reading Assignment: Sec 3.1-3.2 of Introduction to Applied *LA* (third reference book)

Tutorial Questions: Qns 3.3-3.5 of Introduction to Applied *LA* (third reference book)

Stage 4: Optimization

Directional Derivative (vector input - scalar output)
Innerproduct of Directional derivative with a unit vector (= Gradient along that direction)
Steepest descent using Directional derivative
Jacobian Matrix (vector input - vector output)
Backpropagation using Jacobian and Directional derivative

Optimization

Reference for Directional Derivative

Reference for Backpropagation

Solved Tutorial: Solved examples 1,2,3,5,6 in **the Reference for Directional derivative**

Class Test -1

Class-test 1 Question Paper

Class Test 1 Marks

Programming Assignment 1

Extra Class "Intro to Python" (Sep 9, 3:00 PM): For students new to Python (to facilitate Program assignments).

Intro to Python and Google Collab
Object Oriented Programming in Python
Intro to Numpy, Pandas and Matplotlib
Intro to Linear Algebra tools in Numpy

Intro_to_python

Program Assignment 1

Instructions for Submission and Mark Distribution

Mid-Semester Exam

Midsem Marks